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Quasiconformal Mappings and their Applications

Overview

The main purpose of the course is to present the basical principles and concepts of the theory of Quasiconformal Mappings, and to promote the activity of young scientists working (planning to work) in this topic. The course will, for the most part, focus on the following:

- notion of Quasiconformal mappings, analytical and geometrical approaches to the definition of Quasiconformal mappings;
- extremal length and moduli of curve families;
- local and global properties of Quasiconformal mappings;
- theorem of existence of Quasiconformal mapping with given dilatation;
- boundary behavior of Quasiconformal mappings;
- Quasiconformal mappings of Riemann surfaces and Teuchmuller theory;
- locally-Quasiconformal mappings;
- application of Quasiconformal mappings and numerical methods.

The relevant themes from Quasiconformal Mappings will be introduced to the participants through a series of lectures/tutorials on this topic. The tutorial hours will mostly consist of problem discussion sessions. These lectures are intended to serve as a catalyst for gaining basic/fundamental knowledge and generating new ideas for further collaborative activities. The course is aimed on master students acquainted with basic courses od Real, Complex and Functional Analysis.

Objectives

The primary objectives of the course are as follows:

1. Exposing young participants to the fundamentals of quasiconformal mappings and their applications.
2. Building in confidence and capability amongst the participants in the area of quasiconformal mappings.
3. Providing exposure to problems and their solutions, through live discussions.
4. Enhancing the capability of the participants to identify recent research topics in quasiconformal mappings for further studies and research.

Course details

Module A: Quasiconformal Mappings in Plane

Lecture 1: Univalent mappings in the plane: classification, problems, key definitions, examples.

Lecture 2: Definition of Quasiconformal diffeomorphisms in the plane. Characteristics, examples. Comparison with Conformal case.

Tutorial 1: Problem discussion session based on Lectures 1 & 2

Lecture 3: Problem of Grotzsch. Its solution and generalizations.

Lecture 4: Local properties of Quasiconformal mappings.

Tutorial 2: Problem discussion session based on Lectures 3 & 4

Lecture 5: Composition of Quasiconformal mappings, mapping inverse to Quasiconformal.

Lecture 6: Extremal length of families of curves. Definition, examples.

Tutorial 3: Problem discussion session based on Lectures 5 & 6

Lecture 7: Properties of extremal length of families of curves.

Lecture 8: Grotzsh's lemmas for extremal lengths. Generalizations of extremal length method.

Tutorial 4: Problem discussion session based on Lectures 7 & 8

Lecture 9: Geometrical approach to definition of Quasiconformal homeomorphisms. Basical properties of Quasiconformal homeomorphisms.

Lecture 10: Boundary behavior, Ahlfors-Beurling condition. Quasiconformal extension and Schwarzian derivative.

Tutorial 5: Problem discussion session based on Lectures 9 & 10

Module B: Quasiconformal Mappings (Contd) with applications

Lecture 1: Consequences from Green's formulae. Formulae of Borel-Pompeiu.

Lecture 2: Generalized derivatives, equivalent definitions.

Tutorial 1: Problem discussion session based on Lectures 1 & 2

Lecture 3: Analytical approach to definition of Quasiconformal homeomorphisms. Beltrami equation.

Lecture 4: Theorem of existence and uniqueness of Quasiconformal mappings with given dilatation: normal form of solution of Beltrami equation.

Tutorial 2: Problem discussion session based on Lectures 3 & 4

Lecture 5: Theorem of existence and uniqueness of Quasiconformal mappings with given dilatation (contd): existence of solution.

Lecture 6: Theorem of existence and uniqueness of Quasiconformal mappings with given dilatation (contd): univalence of solution.

Tutorial 3: Problem discussion session based on Lectures 5 & 6

Lecture 7: Riemann surfaces: key definitions.

Lecture 8: Quasiconformal mappings of Riemann surfaces.

Tutorial 4: Problem discussion session based on Lectures 7 & 8

Lecture 9: Teuchmuller spaces: key concepts.

Lecture 10: Numerical modeling of Quasiconformal mappings.

Tutorial 5: Problem discussion session based on Lectures 9 & 10